Exercise 1.3: The Black Box: How Science Works (*Grades 3-12*)

What is **science**? It can be defined as any approach that involves the gaining of knowledge to explain the natural world. The scientist tests ideas by gathering evidence. A popular opinion of science has been that it is done in a very specific way, always following a set number and order of steps. This is not the case. Science actually is a very creative endeavor and consists of the interaction of elements, such as exploration and discovery, community interaction, and contributions to society, while still maintaining the central notion of testing ideas. For an illustration of the ways in which these elements of science interact, see Figure 1 below offered by the University of California Museum of Paleontology at UC Berkeley. Visit their site for a more detailed examination of the interactions of the areas depicted in the figure.

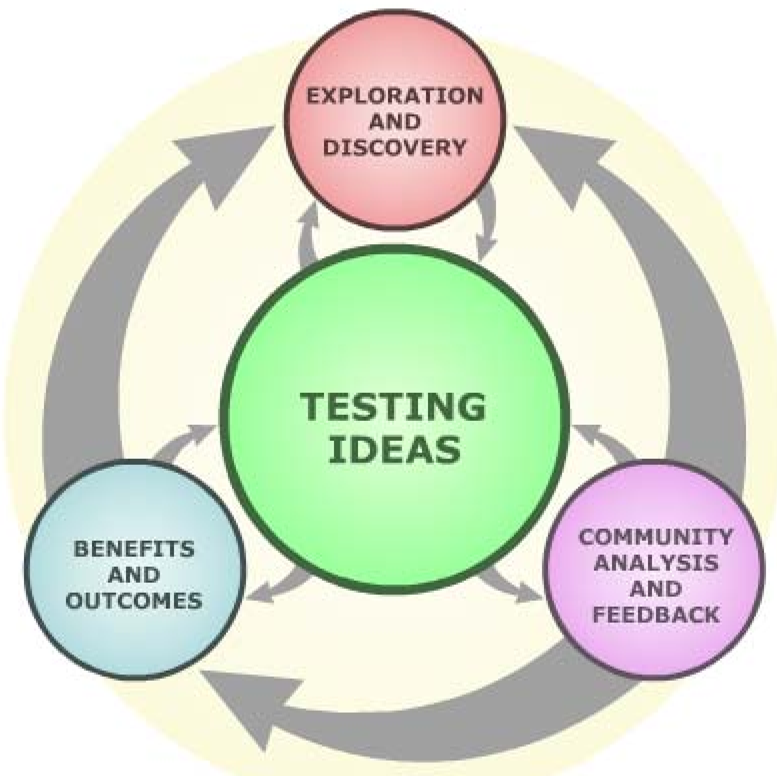

Fig. 1 Interaction of the various elements of science, including the central focus of testing ideas.
Understanding Science. 2012. University of California Museum of Paleontology. 3 January 2012 <http://www.understandingscience.org>

In this series of exercises, collectively referred to as 'Black Box' experiments, you will first focus on the fundamental principle of testing ideas (see Fig. 2 below), but expand your view of science, and how other different elements of science (such as discovery, community interaction, and benefits to society) can interact and contribute to the exciting field of science as a whole.

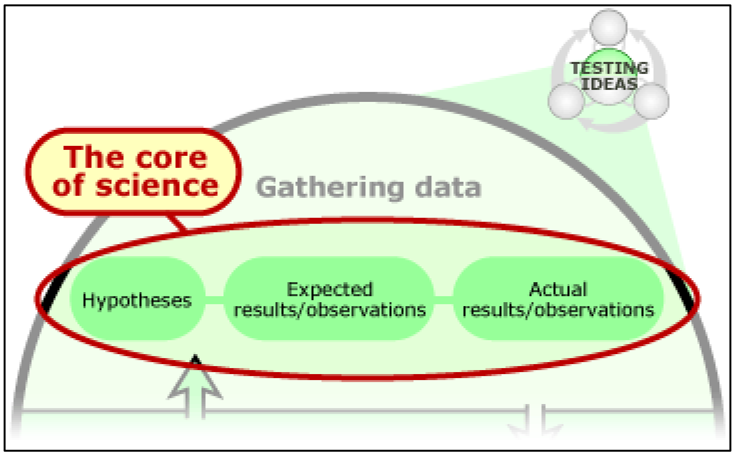

Figure 2. Hypothesis testing. Understanding Science. 2012. University of California Museum of Paleontology. 3 January 2012 <http://www.understandingscience.org>

When it comes to testing ideas, you will often hear about the testing of a particular hypothesis. Hypotheses are referred to as "educated guesses," which are potential explanations of a particular natural phenomenon. For example, let's say you have two flower beds in which you have planted seeds from the same seed packet. One flower bed is in your front yard and the other in your back yard. When you examine the plants that have been produced by the seeds, you observe that those in the front yard flower bed are much taller than those in the back yard. You could come up with several possible explanations (hypotheses) as to ***why*** this might be the case. For example, you might argue that the flower bed in the front yard gets more sun, gets or retains more moisture, has better soil quality, or even some combination of these and other factors not listed. Each of these possible explanations is an idea that could be tested.

Let's assume, that you decide to test your hypothesis that the flowers in the front yard are taller because you think the front yard receives more light. What kinds of evidence would you gather to test this idea and would you make an assumption about some aspect of your hypothesis that you will not be testing directly? (By assumption, we refer to something that is accepted as fact without proving it first hand). Well, first, you could do something simple, such as placing light meters in each flower bed to record the amount and intensity of light that each received throughout a particular time period. If in comparing the light records between flower beds you find that the light meter readings from the front yard bed reflect, on average greater light exposure than those collected from the back yard bed, then your hypothesis is supported.

The assumption you might then make based on your knowledge of the factors that influence plant growth is that increased light is contributing to the production of taller plants in the front yard. You could actually test this assumption directly by setting up an experiment with a new package of seeds of that flower species, growing them under different light levels. In this case, you would predict, before the flowers started to grow, that the seeds exposed to more light would grow taller.

In the two experiments described above, you would be gathering evidence to test your hypothesis that light exposure differences may lead to differences in plant growth. The results of tests of ideas may either support or fail to support a particular hypothesis. If the results fail to support the original hypothesis, a new, revised hypothesis can then be formed and tested. If, however, the hypothesis is supported by the results, that doesn't necessarily mean a stopping point has been reached. Just because: 1) there is a correlation between the amount of light the two sections of your yard receives and plant height; and 2) that light contributes to plant growth does not necessarily mean that different light levels has caused the difference in plant heights you observed between your front and back yards. Some other factor such as differences in soil composition or frequency differences in the watering of the two flower beds might be the prominent contributing factor.

When hypotheses are continuously supported when subjected to very many tests, may become part of a **scientific theory**. It is very important to note that non-scientists often use the word "theory" to mean "just a guess." However, in science, the word "theory" has a very different, very specific meaning: ***"a well-substantiated explanation of some aspect of the natural world that can incorporate facts, laws, inferences, and tested hypotheses"*** (National Center for Science Education 2008).

In the following series of exercises, you will utilize various elements of science, as well as the laws of probability to help you decide on the possible identity of one or more mystery objects (from a set of 11 potential objects) in a container (the “Black Box”) without opening the container first. In science, the term “black box” is often used to refer to something unknown.

Under exercise 1.3A, you will use the senses available to you in formulating hypotheses (ideas) about the identity of the object(s) in the Black Box. To assist you in this endeavor, this unit also has another container, the “White Box,” which you may open. The White Box contains a series of objects from which the mystery object(s) in the Black Box have been selected: a toothpick, crayon, paper clip, rubber ball, metal ball, cotton ball, rubber band, marble, penny, poker chip, and a cork. Depending on your grade level, you will be asked to either attempt to find the identity of one (grades 4-5) or two (grades 6-12) items in the Black Box. If it contains two objects, however, it will contain two *different* objects, and never two objects of the same type.

In Exercise 1.3B, you will compare your success in identifying the two objects through the use of the scientific principle of testing ideas versus the success you would have in merely guessing what is in the Black Box. You will apply the rules of mathematical probability in determining what number of guesses would be required to correctly guess the identity of the unknown item (s).

Exercises 1.3C & 1.3D are further extensions of the previous exercises, and are intended to provide you with an even more realistic understanding of how science works than the illustrations provided by Exercises 1.3A &B.

ONE RULE APPLIES THROUGH ALL OF THESE EXERCISES: NO PEEKING INTO THE BLACK BOX UNTIL YOU COMMIT YOUR HYPOTHESIS TO PAPER, ANNOUNCE IT TO YOUR CLASS, AND YOU ARE INSTRUCTED TO OPEN IT!

Exercise 1.3A.1: Black Box Trial 1: Exploration & Testing Ideas (*Grades 4-5*)

NOTE TO TEACHERS: Make sure each "Black Box 1.3A" tin has only *one* item inside it for grades 4-5, but *two different* items for grades 6-12.

- Divide into teams. (Six sets of this experiment are available). Each team should have a container labeled as the "White Box", a container labeled "Black Box 1.3A," and an empty clear plastic container. **NOTE: The Black Box container should not be opened during the course of this experiment.**
- Open the "White Box" container. Check to see that there are 11 unique items in this container. If not, contact your teacher for replacements. **DO NOT TOUCH OR OTHERWISE INTERACT WITH YOUR GROUP'S BLACK BOX AT THIS POINT!**
- Examine each of the 11 objects in the white box, paying particular attention to their geometric attributes (shapes). Note whether each object is spherical (ball-shaped), flat, rectangular, round, etc. Try to think of other characteristics that might help you determine the identity of the mystery item in the Black Box

without first opening it! How about the sound an object makes in the tin container (soft or loud, a thud or a clink, etc.), how heavy it feels in your hand relative to other objects etc. You may wish to make observations of the characteristics of specific items placed in the White Box one at a time to give you an idea on what you might observe in the Black Box later. If you do so, you can place all the other White Box items in the clear plastic container to keep them from rolling off your desk. Be creative in devising a testing plan! What characteristics will you examine and how will you do this?

- Make a table listing each of the 11 objects as rows in a table like the one shown below for comparing US coins. The column headings are characteristics or attributes that you have recorded for each coin type. For instance a penny is round and flat (characteristic 1), coppery in color (characteristic 2) and the third largest of the coins listed (characteristic 3).

Item	**Characteristic 1 Shape**	**Characteristic 2 Color**	**Characteristic 3 Size (1 = largest)**	**Etc and so on**
Penny	Round, flat	Coppery	3	
Nickel	Round, flat	Silver	2	
Dime	Round, flat	Silver	4	
Quarter	Round, flat	Silver	1	

- Examine the table that you have constructed that lists the characteristics of each of the potential items from the White Box that could be in your Black Box. You have just used one element of science (exploration) to explain some aspect of the natural world (the identity of an object in your Black Box).
- Now take a moment to again think about the central focus of science which is to test ideas. You know that there are actually 11 different objects that could possibly be in the Black Box. Therefore, there are a total of 11 different ideas (hypotheses) that you can test!
- Make a list of all 11 possible hypotheses for this experiment, each in the form "There is a ________ in the Black Box."
- Beside each hypothesis, make one or more predictions as to what you might observe about the Black Box (again, without opening it!) if that hypothesis is correct, using some of the characteristics you used in the table you have constructed.
- For example, if the particular hypothesis is "There is a cotton ball in the Black Box," and the traits you used were the sound and perceived weight of the 11

objects from the White Box, a logical prediction might be “If there is a cotton ball in the Black Box, the Black Box will be very light, and not make a noise when shaken.”

- After you have made a list of all possible hypotheses and predictions based on the characteristics you used in constructing your table, it is time to put each of these hypotheses to the test!
- Using the cotton ball example above, you could test your prediction by picking up the Black Box and shaking it. If the Black Box does not feel very heavy, and does not make a sound, that supports your initial hypothesis that a cotton ball is in the unknown item within. However, be aware that there may be other hypotheses regarding the contents that would have similar predictions!
- If when you picked up the Black Box, it felt heavy and made a clanging sound when shaken, this does not match your initial prediction. You must then revise your hypothesis. Using your table of attributes of each possible object, as well as your list of hypotheses and predictions, draw a line through each hypothesis whose predictions are not met by your tests on the Black Box. You should be able to eliminate several hypotheses in this way.

- Did you have more than one hypothesis that was still supported after your tests on the Black Box? If so, examine all the White Box items again, and see if you can come up with any other traits that might help you further narrow down the possible identity of the Black Box object without opening it yet.
- In the end, decide as a team, what the object within the Black Box may be and prepare your statement as to the evidence that you have gathered in support of this hypothesis. DO NOT OPEN THE BLACK BOX YET!
- After all teams have come to a decision on the identity of their mystery objects, your teacher will have each team tell the class what your conclusion is and how you came to arrive at that conclusion.
- Your team will then open your Black Box. Were you correct?
- One of you or your teacher should record on the board the number of teams that correctly identified the objects in their Black Boxes, as you will use this information in comparing the probability of obtaining a correct answer to a question posed through use of scientific practices versus simply guessing under **Exercise 1.3B1.**
- Go on to this latter exercise.

Exercise 1.3A2: Black Box Trial 1: Exploration & Testing Ideas (*Grades 6-12*)

NOTE TO TEACHERS: Make sure that each "Black Box 1.3A" tin has TWO different items from the 11 possible items inside it when using this exercise for grades 6-12. Also check to see that the White tin has the following 11 unique items in it: a toothpick, crayon, paper clip, rubber ball, metal ball, cotton ball, rubber band, marble, penny, poker chip, and a cork. There are six sets of this exercise available.

- Split into small groups of students. All of the students within each 'team' formed will collaborate to determine which two mystery objects they have received.
- Each group should be given one tin labeled as the White Box, a second tin labeled "Black Box 1.3A", and one of the clear plastic containers (which can be used to keep White Box objects not in use from rolling off of desks/tables). **DO NOT TOUCH OR OTHERWISE INTERACT WITH YOUR GROUP'S BLACK BOX AT THIS POINT!**
- Refer back to the diagram in Figure 2 that illustrates the elements of science with respect to testing ideas.
- Examine the objects in the White Box, listing them on a sheet of paper or on the board at the front of the room so that you may consult the list as your investigation proceeds.
- As a group, you should consider how you will approach the problem scientifically, using the scientific element of exploration. For example you might choose to explore the characteristics (sounds, mass) of the Black Box itself and record these. Your hypothesis would be that the objects would have to have a particular sound, weight, and/or behavior relative to one another. Examples of alternative approaches include exploring the behavior of the 11 objects, individually, or in pairs in the White Box tin, and then making predictions about how the objects should sound etc.) if they were in the Black Box.
- If you need help in defining your approach, you can consult the steps described under the answers for Exercise 1.3A. However, you will learn more from devising the technique you apply to the problem yourselves.
- Write the testing procedures you plan to use on a sheet of paper.
- Start the process, remembering to keep in mind the central principle of testing ideas (Figure 2)!
- As you gain information through exploration, begin making a list of hypotheses and assumptions under each of the hypotheses as you progress.
- Record the steps you have taken throughout your investigation.

- Finally record what your team concludes is in the Black Box, **but do not open the Black Box yet!**
- When finished, each team should report the following information to the class:
    - the approach they have taken to the problem,
    - the hypotheses they constructed,
    - the steps they used to test their hypotheses,
    - revisions to hypotheses inspired by the testing process,
    - and finally, what two objects they feel are in the Black Box and why.
- Now the team should open the Black Box and check to see if they were correct.
- Tally the number of teams that correctly obtained the two items present in their Black Box versus the number of teams that failed to obtain the identity of both items on the board for the class to see.
- Calculate the proportion of teams that correctly identified the two unknown objects. If three of the six teams obtained the correct pair of items, your success would have been 3/6 =1/2 = .5 = 50%.
- Use this information in comparing the probability of obtaining a correct answer to a question posed through use of scientific practices versus simply guessing under the rules of probability in **Exercise 1.3B2.**
- Go on to this latter exercise.

Exercise 1.3B.1: Using Simple Probability (*Grades 4-5*)

Suppose that instead of using the exploration element of science to assist you in testing ideas, you just ***guessed*** which item was in your Black Box without interacting with it in any way. Would you have done as well? Probably not! The reason is that, although using the exploration element of science to inspire ways of testing the identity of the objects in the Black Box may not have *completely* determined which objects were in the Black Box, it helped you to narrow the list of possibilities. On the other hand, guessing involves chance. When we talk about the chance that a particular event will happen, we are talking about **probability**. Thus when we hear about the chance of rain, the chance of snow, or the chance that any particular weather event will happen, we are hearing about the *probability* of a particular weather condition occurring. We also hear about the chance of winning the lottery, which is really the *probability* that we will win, or how likely we are to win.

Making predictions about the likelihood of a certain event happening is a very important skill, and helps us in many areas of our lives. Aside from just being useful information about the weather and the lottery, understanding probability is very useful in studying health and disease, transportation safety, sports, and protecting the environment, just to name a few.

First, it is important to understand the distinction between two similar terms that are often (incorrectly) used interchangeably: probability and odds. The **probability** of a certain occurrence can be expressed in many ways, such as a fraction, a decimal, or a percentage, and represents the chances for that particular occurrence divided by the total chances of any occurrence. For example, imagine rolling a single die. If you wanted to know the probability of rolling an even number, this would be calculated as follows. An even number could be rolled 3 different ways (as a 2, 4, or 6). There are a total of six different results that could be rolled, however. So, the probability of rolling an even number would be equal to

$$\boldsymbol{Probability\ of\ rolling\ an\ even\ \#\ on\ die} = \text{"3 out of 6"} = \mathbf{3:6} = \frac{\mathbf{3}}{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{2}} = \mathbf{0.5} = \mathbf{50}\%$$

- Now choose a characteristic or attribute that one or more of the 11 items in the white box possesses. Find the probability of an object possessing that characteristic being in the Black Box.
- Express this probability in the five different forms it can be expressed as shown in the equality equation above.
- Working with your team, repeat this process with other attributes to find the probability of objects possessing a particular attribute of being present in the Black Box.
- Be sure to record these results in a table.

You have just determined the *simple probability* of objects with certain characteristics being in the Black Box!

You have probably heard people talk about **odds**. Odds are related to probability, but with a distinct difference. *Probability* compares how likely it is a certain event will happen compared to the total number of possibilities, while *odds* are the comparison of the number of favorable outcomes (what you are looking or hoping for) to the number of unfavorable outcomes (what you are <u>not</u> hoping or looking for). Going back to our example of rolling a die, the probability of rolling an even number is 3:6, because there are three possible even results out of the total six possible results.

On the other hand, the odds of rolling an even number are 3:3 because there are three possible even numbers that could be rolled (2, 4, & 6), and three possible results that are *not* even numbers (1, 3, & 5).

Now let's apply this knowledge to your Black Box problem.

- Find the *odds* of an object with a particular characteristic being in the Black Box.

Q1. What is the probability of spherical (ball-shaped) object being in the Black Box? Express this probability in the five different ways described above. Remember, all of these values mean exactly the same thing!

Q2. What are the **odds** of a spherical object being in the Black Box?

Q3. If you were to simply guess the identity of the object in your Black Box without using elements of science to guide your interaction with the Black Box, what would be the **probability** that you got the answer correct?

- Examine the proportion of groups that arrived at the correct identity of the objects in Black Box 1.3A, and compare this to the answer to the previous question.

It is very likely that the proportion of groups that got the correct answer is larger than the probability obtained by guessing alone. This is because, by using the elements of science to guide you in testing ideas, you were in effect narrowing down the list of possible objects in the Black Box, thus increasing the probability that you got the answer right!

Check your answers to these questions in the Answers section of this book, under Exercise 1.3B.1!

Exercise 1.3B2: The Rules of Probability (*Grades 6-12*)

Suppose that instead of using elements of science to help you test ideas, you just guessed which items were in your tin without performing any experiments or observations. Would you have done as well? Probably not! The reason is that, although using elements of science may not have *completely* determined which objects were in your tin, it helped you to *narrow the list of possibilities*. Thus, it *increased the probability that your final choices would be correct*. In this exercise, we will examine the laws of probability, which will mathematically explain why applying scientific principles to the Black Box problem beats merely guessing the contents.

First, it is important to learn a bit of terminology involving probability. To every experiment, there corresponds a set of *possible outcomes*, called the **sample space** of the experiment. For example, the sample space of rolling a single die can be represented as follows: {1,2,3,4,5,6}, with each of the numbers in the brackets representing all of the possible results of the roll. A subset of the sample space is called an **event**. To be more specific, an event is a set that contains some (possibly all) of an experiment's outcomes. For example, the set $\boldsymbol{E} = \{\mathbf{2}, \mathbf{4}, \mathbf{6}\}$ is an event, representing rolling an even number on the die. Note that the order in which we list the elements in the set does not matter. This means that the sets $\{\mathbf{2}, \mathbf{4}, \mathbf{6}\}$ and $\{\mathbf{6}, \mathbf{2}, \mathbf{4}\}$ (as well as all possible orderings of 2, 4, and 6 in a set) are considered to be the same.

Events such as $\{\mathbf{2}\}$, which contain a *single element* are called **elementary events**. If two events contain none of the same elementary events, then they are said to be **mutually exclusive events**. For example, $\{\mathbf{2}\}$ and $\{\mathbf{4}, \mathbf{6}\}$ are mutually exclusive events.

Q4. Suppose that you roll a six sided die. Let $\boldsymbol{E}$ denote the event that you roll less than a five. Write down all of the elements that belong to the event $\boldsymbol{E}$.

Q5. Let $\boldsymbol{B}$ be the event that you roll $\mathbf{1}, \mathbf{4}$, or $\mathbf{6}$, that is, let $\boldsymbol{B} = \{\mathbf{1}, \mathbf{4}, \mathbf{6}\}$. Are $\boldsymbol{B}$ and $\boldsymbol{E}$ mutually exclusive? If not, which elementary events belong to both $\boldsymbol{B}$ and $\boldsymbol{E}$?

However, what is probability itself? The **probability** that any specific event occurs is a measure of the likelihood that the event will occur. The probability of a certain occurrence can be expressed in many ways, such as a fraction, a decimal, or a percentage, and represents the chances for that particular occurrence divided by the total chances of any occurrence. If $\boldsymbol{E}$ is an event, then we denote the probability that $\boldsymbol{E}$ occurs by $\boldsymbol{P}(\boldsymbol{E})$. Sometimes the probability that an event occurs can be determined through intuition. At other times it may be determined experimentally. There are three basic **axioms** (rules) of probability that are used to determine the probability that an event occurs. On the following page, you will find a list of these basic axioms, expressed in both a mathematical format, as well as explained in words. Understanding these basic axioms, as well as further rules that are extensions of these axioms will assist you in solving problems using probability.

Axioms of Probability

1) If $\boldsymbol{S}$ is the sample space of an experiment, then $\boldsymbol{P(S) = 1}$
2) If $\boldsymbol{E}$ is any event, then $\boldsymbol{0 \leq P(E) \leq 1}$
3) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually exclusive events, then $\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$

Or, expressed verbally,

1) The sample space *S* of an experiment is the set of all possible outcomes. One of the outcomes in the sample space will definitely occur.
2) The likelihood of a particular event ranges from impossible to absolutely definite. (Probabilities are usually expressed as fractions, or more commonly, decimals, ranging from 0, or 0% likely, to 1, or multiplied by 100 for expression as percents as in 0 to100% likely.)
3) If an outcome can be one of two alternatives (but not both), the probability of either event occurring is equal to the sum of the likelihood of each event's occurrence.

A couple of very useful rules follow directly from the axioms of probability, which we will discuss below.

Rule 1:
When every outcome in a set of possible outcomes is equally likely to occur, the probability that a specific outcome occurs is equal to one divided by the number of possible outcomes.

For example, imagine rolling a single die. Since a fair die should have equal surface areas on each of its faces, obtaining any result should be equally likely. For example, the probability of rolling a 1 is the same as the probability of rolling a 2, a 3, a 4, a 5, or a 6. Since there are six possible outcomes, the probability of obtaining any of these results would thus be equal to 1/6 (which is approximately equal to 0.167, or 16.7%). In any case, the probability of an event is equal to the number of ways that the event could happen divided by the total possible results. For example, if you wanted to know the probability of rolling an even number on a fair die, this would be calculated as follows. An even number could be rolled 3 different ways (a 2, 4, or 6). There are a total of six different results that could be rolled, however. So, the probability of rolling an even number would be equal to

$$\textit{The probability of rolling an even \# on a die} = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

This is an example of a **simple theoretical probability**, which again is the proportion of a particular possible outcome out of the total number of possible outcomes.

However, since your Black Box has two objects in it, there are separate probabilities for each object in your Black Box having a particular identity. However, when there are separate probabilities for different events, these probabilities can be combined into a single **compound probability**. For example, consider the outcome of rolling a single die and flipping a coin. The probability of obtaining an even number on the die would be equal to 1/2 (= 0.5 = 50%), and the chance of obtaining a result of tails on the coin would also be 1/2. However, what if we wanted to know the probability of obtaining an even number on the die *and* a result of tails on the coin? In order to calculate the compound probability of two events, first we should be aware of whether the events are **independent**. **Two events are independent if the occurrence of one event does not affect the probability that the other event will occur.** Our die and coin example is a good example of independent events, because the result of the die roll has no influence on the result of the coin flip. The contents of each group's Black Box 1.3A containers are also good examples of independent events. Since the pair of objects in each Black Box 1.3A is determined randomly, the contents of one group's Black Box 1.3A has no effect on the contents of any other group's Black Box 1.3A.

When two events are independent, the probability that both events occur is equal to the product of the probabilities that each event occurs.

- **In other words, if A and B are independent events, then $P(A\ and\ B) = P(A)P(B)$.**
- **Another way of saying this is that to find the compound probability of two independent events, multiply the probability of the first event by the probability of the second event.**
- **In fact, the converse is also true. If the probability that two events occur is equal to the product of the probabilities that each event occurs, then the events are independent.**
- **If $P(A\ and\ B) = P(A)P(B)$, then A and B are independent events.**

Dependent events, on the other hand, are events in which the outcome of one event <u>does</u> have an effect on the outcome of the second event. For example, imagine a bag that holds 8 pieces of candy: 3 green and 5 red. The probability of getting a red piece on the first draw is 5 out of 8, but with each consecutive draw, the probability of getting a certain color changes because the total number of pieces decreases as you continue to draw pieces out of the bag. The probability of getting a certain color *depends* on how many and what color candies have already been removed from the bag. To find the probability of dependent events, find the probability of the first event, then find the probability of the second event from which you have removed the first event. In the above bag of candy, the probability of choosing a red candy on the first draw is 5 out of 8, the probability of getting a green candy on the second draw *without returning the red candy to the bag* is 3 out of 7 because there are now only a total of 7 pieces in the bag. Multiply the two probabilities together to get a compound probability of 15 out of 56.

The probabilities of each object's identity in a single group's Black Box 1.3A is dependent on the identity of the other object. The probability of the first item being a crayon in the Black Box is equal to 1/11. But, because the two objects in your group's Black Box 1.3A are different, if the first object in your Black Box 1.3A is a crayon, the probability that the second object is a crayon is equal to zero. It has to be a different object. Likewise, if one object in the box is NOT a crayon, the probability of the other object being a crayon is equal to 1/10 (the second object can only be 1 of 10 objects, since it has to be different than the first).

- Now answer the following questions about independent/dependent events.

Q6. Suppose two coins are tossed. Let ***A*** be the event that the first coin is heads, and ***B*** be the event that the second coin is heads. Are ***A*** and ***B*** independent?

Q7. Suppose that two children attend the same daycare. Let ***A*** be the event that the first child catches a cold and ***B*** be the event that the second child catches a cold. Are the events ***A*** and ***B*** independent?

We can use the ***counting principle*** to help us solve for the total possible outcomes of two events.

The Counting Principle

If there are x ways to perform one task, and y ways to perform a second task, then there are xy ways to perform both tasks.

Suppose for example that we flip two coins. There are 2 ways to flip the first coin (heads or tails) and 2 ways to flip the second coin, so there are 2×2 = 4 (possible outcomes when we flip both coins) ways to flip both coins.

In our Black Box, how many outcomes are possible? In this case, the possible outcomes are the possible pairs of objects that could be present in the Black Box. Since the pairs of objects in the Black Box are determined randomly, any pair of objects is equally likely. For example, the probability that the Black Box contains a marble and a cork (or any other pair of objects) is equal to one divided by the number of possible pairs.

What would be the probability of correctly guessing that the Black Box contains a marble and a cork? To answer this, we first need to count the number of possible pairs. We'll start by counting the number of ways there are to form a pair.

In our case of 11 objects, there are eleven ways to choose the first object, but only ten unique ways to add the second member of the pair, since each Black Box tin will contain two *different* objects. Therefore, according to the counting principle, there 11 × 10, or 110 ways to form a pair.

- Assume that instead of using elements of science to determine which pair of objects is in your Black Box, you randomly choose a pair that contains two of the eleven possible objects and then guess that this is the pair in your Black Box without interacting with it in any way. Since you choose the pair at random, you are equally likely to choose any pair. This means that you are just as likely to choose a toothpick and a cotton ball as you are to choose a marble and a cork.
- So, what is the probability that you choose a marble and a cork? The answer to this question depends on the following basic rule of probability:
- To test your understanding of Rule 1 and the Counting Principle, answer the following two questions:

Q8. Suppose that you have a marble, a metal ball, and a penny. Imagine that you form a pair by choosing two objects from this set. How many ways can you form a pair?

Q9. How many *distinct* pairs can be formed from the three items above?

- Check your answers to these questions in the Answers section of this book, under Exercise 1.3B.2.

Although there are 110 different ways to form a pair from the set of eleven objects present in the White Box in our experiment, there are actually only 55 different pairs. This is because in the scenario we described under **Q9** above, there are two ways to form every pair. For example, we could form the pair with a metal ball and a rubber ball by choosing the metal ball and then the rubber ball, or by choosing the rubber ball and then the metal ball. Thus, it follows that there are half as many pairs as there are ways to form a pair (110/2 = 55).

Now we can figure out the probability of getting the identity of both objects in the Black Box correct by guessing alone, without using any elements of science. Since there are 55 different pairs of the 11 objects in the White Box, and you are equally likely to choose each pair, the probability that you choose a marble and a cork is $\frac{1}{55}$. That is, you make approximately 2 correct guesses for every 100 guesses you make. Thus, you can see that the probability of getting the identity of your mystery objects correct by guessing alone is very low!

- Compare this probability to the proportion of Black Box pairs your teams correctly identified.
- Now ponder the following question: What is the probability that you choose a pair that has a penny?

In order to answer to this question, remember our definition of an event. An ***event*** is a set of outcomes. Here the outcomes are pairs of objects, and the event of interest is a set of pairs that have pennies. Any pair that has a penny is in this event.

Q10. How many pairs in our Black Box experiment are in an event that has a penny?

- The answer to **Q10** depends on another basic rule of probability shown below (Rule 2). Apply this rule and check your answer in the Answers section under Exercise 1.3B.2.

Rule 2:
When every outcome in a set of possible outcomes is equally likely to occur, the probability that a specific event occurs is equal to the number of outcomes in the event divided by the number of possible outcomes.

- Test your understanding of probability theory by tackling the following related questions.

  Q11. What is the probability that your Black Box has a pair consisting of a penny or a marble, but not both a marble and a penny?

  Q12. What is the probability that, by guessing alone, you correctly guess the identity of one (but not both) of the objects in your Black Box?

Combinations, Permutations, & Ways of Selecting Objects

In the previous exercise and questions, it was explained that there are 110 different ways to **form** a pair of objects in the Black Box. This is because you know that there are no two identical objects in the Black Box. Thus, though there may be 11 choices for the first item placed in the Black Box, that leaves only 10 choices for the second item, and thus there are 110 ($11 \times 10 = 110$) different **ways** in which two objects could have been placed in the Black Box. However, the question of interest is what two objects are in the Black Box, and not the order in which those objects were placed into the Black Box (which you would not be able to determine). For example, you may think that you have a marble and a penny in your Black Box. However, this pair of items could have been constructed in two ways. Your teacher could have placed the penny in first and then the marble, or the marble first and then the penny. Either way still results in the same pair of objects, so the number of **possible pairs** of objects is only half the number of the number of **ways in which pairs could be constructed**.

Another way to handle similar tricky probability problems like these would be to think about the concepts of **combinations** and **permutations**. Both combinations and permutations involve groups of objects, numbers, etc. However, the major difference between the two is whether or not the **order** of the objects/numbers is important. In this exercise, you will learn a little more about both combinations and permutations, in cases where repetition is/is not allowed, and some formulas related to each scenario that can be used to make probability problems easier.

A **combination** is a group of objects/numbers in which order ***is not*** important, and a **permutation** is a group of objects/numbers in which order ***is*** important. For a fun (and tasty) example, let's consider a trip to an ice cream shop. In our hypothetical ice cream shop, 5 flavors of ice cream are available: chocolate, vanilla, strawberry, banana, and cherry. This particular ice cream shop only offers two options: milkshakes and cones, both of which come in sizes ranging from one to five scoops.

A milkshake at this shop would be the equivalent of a **combination**, since the order in which flavors are added to the blender wouldn't matter, since they all get blended together.

A cone in this shop, however, is an example of a **permutation**, where order *does* matter, since you would be eating the scoops of different flavors in a particular order from top to bottom (and you might like to eat a certain flavor first or last!).

- Take a moment and see if you can write down all the different flavor **combinations** for a three-scoop shake, if you do not use more than one scoop of any flavor.

You should have determined that there are a total of 10 different three-scoop shakes (combinations) that you can get if you do not use more than one scoop of any flavor:

- vanilla, chocolate, strawberry
- vanilla, chocolate, banana
- vanilla, chocolate, cherry
- vanilla, strawberry, banana
- vanilla, strawberry, cherry
- vanilla, banana, cherry
- chocolate, strawberry, banana
- chocolate, strawberry, cherry
- chocolate, banana, cherry
- strawberry, banana, cherry

You might think that it would be very difficult to figure out how many different shakes or cones of a particular size you would be able to get at this shop if they added a lot more flavors, or expanded the maximum size (number of scoops) allowable in a shake or on a cone, particularly if you also consider whether or not duplicate scoops of flavors are involved. However, there are mathematical formulas that can help you find these answers. Each of these formulas involves two variables: the number of objects from which a choice can be made (***n***), and the number of choices that are made (***r***).

Three of these formulas also involve the use of **factorials**. A **factorial** is a particular mathematical function of positive integers (whole numbers) ***which is equal to the product of all integers less than or equal to the integer in question***. The factorial function is denoted by an exclamation point after a particular integer. For example:

$$\text{"n factorial"} = \boldsymbol{n}! = \boldsymbol{n} \times (\boldsymbol{n}-\mathbf{1}) \times (\boldsymbol{n}-\mathbf{2}) \ldots \times \mathbf{3} \times \mathbf{2} \times \mathbf{1}$$

Factorials for all positive integers are thus calculated in exactly the same way. The only example that may (initially) seem a little strange is the convention that the factorial of zero (**0!**) is considered to be equal to 1. This may be a little more clear if you think about it in a way such that there is exactly one way of arranging zero objects: an empty set.

Where order does **not** matter (like the shakes in our ice cream shop), the following formulas can be used to determine the number of possible **combinations**:

The number of ways to select ***r*** objects from a set of ***n*** distinct objects (when the number of selected objects is less than the total number of objects, or expressed mathematically, $n \geq r$), can be expressed as follows:

$$\textit{Number of combinations (repetition NOT allowed)} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!\,(\boldsymbol{r}!)}$$

The following equation can be used to calculate the number of ways to select ***r*** objects from ***n*** types of objects (when there are at least ***r*** objects of each type available):

$$\textit{Number of combinations (repetition IS allowed)} = \frac{(\boldsymbol{n}+\boldsymbol{r}-\mathbf{1})!}{\boldsymbol{r}!\,(\boldsymbol{n}-\mathbf{1})!}$$

Where order **does** matter (like the cones in our ice cream shop), the following formulas can be used to determine the number of possible **permutations**.

The number of ways, when order is important, to select $\boldsymbol{r}$ objects from a set of $\boldsymbol{n}$ distinct objects (when the number of selected objects is less than the total number of objects, or expressed mathematically, $\boldsymbol{n} \geq \boldsymbol{r}$), can be expressed as follows:

$$\boldsymbol{Number\ of\ permutations\ (repetition\ NOT\ allowed)} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$$

And finally, the number of ways, when order is important and repetition is allowed, to select $\boldsymbol{r}$ choices from $\boldsymbol{n}$ objects, and when there are at least $\boldsymbol{r}$ objects of each type available can be calculated as follows:

$$\boldsymbol{Number\ of\ permutations\ (repetition\ IS\ allowed)} = \boldsymbol{n^r}$$

Using the ice cream shop example in each of these equations, $\boldsymbol{n}$ is the number of possible flavor choices, and $\boldsymbol{r}$ is the number of scoops in our milkshake or on our cone. Therefore, going back to our "three scoop, no flavor duplication" shake example, we can substitute those values into the appropriate **combination** equation as follows (with 5 flavors, taken 3 at a time):

$$\frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!\,(\boldsymbol{r}!)} = \frac{\mathbf{5}!}{(\mathbf{5}-\mathbf{3})!\,(\mathbf{3}!)} = \frac{\mathbf{5}!}{(\mathbf{2}!)(\mathbf{3}!)} = \frac{\mathbf{5}\times\mathbf{4}\times\mathbf{3}\times\mathbf{2}\times\mathbf{1}}{(\mathbf{2}\times\mathbf{1})(\mathbf{3}\times\mathbf{2}\times\mathbf{1})} = \frac{\mathbf{120}}{(\mathbf{2})(\mathbf{6})} = \frac{\mathbf{120}}{\mathbf{12}} = \mathbf{10}$$

- Now answer the following questions:

Q13. Fill in the following table regarding the number of possible shakes and cones in our hypothetical ice cream shop:

# of Scoops	**# of possible shakes (no flavor duplication)**	**# of possible shakes (duplication allowed)**	**# of possible cones (no flavor duplication)**	**# of possible cones (duplication allowed)**
1				
2				
3				
4				
5				

Q14. Regarding the Black Box experiment, are your hypotheses more related to combinations or permutations? Why?

Q15. Using what you now know about combinations and permutations, and the fact that there is NO duplication of objects in the Black Box, calculate the number of different pairs of 11 objects from the White Box that could be placed into a Black Box.

Q16. Since the pairs of items placed into a Black Box are determined at random, what is the probability that there is a marble and a cork in any given Black Box?

Q17. What if you were given a Black Box and told that there were three items in the box (with the same restriction of no identical items). What would be the probability of correctly guessing *all three* items in the box without using the any elements of science, and without any contact with the Black Box whatsoever?

Exercise 1.3C: Black Box Experiment Trial 2: Community Feedback & Analysis (*Grades 6-12*)

Science can never *absolutely* verify something as true. Because you were able to open the Black Box at the end of your investigation in Exercise 1.3A and actually learn what items were in the tin, you obtained a distorted view of the scientific process. This exercise corrects that misconception. In this exercise, all teams will be given a tin labeled "Black Box 1.3C," each of which contains identical pairs of two different objects from the 11 possible items in the White Box. Individual teams should follow the same procedure as that described under Trial 1 (Exercise 1.3A). You will follow the same procedures as before. At the end of this experiment, your teacher will lead a discussion to see if a consensus can be reached amongst all teams as to what two items are present in Black Box 1.3C.

This exercise highlights the importance of another element of science: collaboration and sharing of information within the scientific community, which may not only increase the overall knowledge base, but also help reduce the influence of bias in interpreting one's results (See Fig. 1).

- Each team should obtain a White Box with the 11 potential items present, a clear plastic container and a tin labeled "Black Box 1.3C". These tins will have a screw on them that prevents removal of the lid.

- You may use the data you gathered from Trial 1 in this effort so that you need not start from scratch. In the end, however, be prepared to defend your team's decision with data.
- Complete an initial tally on the board at the front of the room of the team decisions as to what the two items are in the tin.
- Complete some simple statistics on your class data, and/or make a bar graph (e.g., proportions of teams that had the same respective single and pair of items (see tables below for an example of how to present these results.

Team Name	**Pair of Items**	**% teams**
Solutions	Toothpick marble marble toothpick	20%
Sweet Tea	poker chip marble marble poker chip	20%
Avatar	poker chip rubber band rubber band poker chip	20%
No Name	poker chip cotton ball cotton ball poker chip	20%
Complex Numbers	Rubber band marble Marble rubber band	20%
Guess	1/110 1/110 = 2/110 =1/55	1/55 1.8%

Particular item one of the two present	**% teams**
Toothpick	20%
marble	60%
poker chip	60%
rubber band	40%
cotton ball	20%

- If there was no general agreement as to what one or both of the items were in the tins, then each team should present its case for the items they thought were present.
- The teams might want to use techniques others report for themselves in a second go at determining what is in the Black Boxes. In the end at a second column on the board and census the teams again for their decisions.
  Has the community discussion and/or additional examination using new approaches changed the proportion of teams that feel a particular pair of items is in the black box, that a particular item is present? **It is important to understand that in science, no votes are taken to decide what is true.** Dissenting views are not only welcome, but drive further assessment.
- Keep a copy of the class results in preparation for Trial 3.

Exercise 1.3D: Black Box Experiment Trial 3: Outcomes & Benefits. (*Grades 6-12*)

While science cannot *completely* verify something as true, the interaction of the various elements of science often leads to new techniques, tools, and approaches, allowing scientists to have greater confidence in their assessment of questions. You will experience this firsthand in this final exercise.

- Find the container which includes 6 magnets, 6 spring scales, and 6 hair nets (which are used to suspend the White/Black Boxes from the hooks on the spring scales). **NOTE:** Some newer copies of this unit may contain digital scales and calibration masses instead of spring scales and hair nets.
- Each team should take one of each of these items. These represent new techniques or technologies that were previously unavailable in your earlier experimentation with the items in Black Box 1.3C under Exercise 1.3C. The provision of new tools to you in this exercise, parallels the development of new techniques or tools in many scientific fields. These new approaches may have been developed based on inspiration from other scientific work or even result from arguments made by scientists with dissenting opinions concerning the consensus view of prior testing of an idea.
- You can now use these tools to assist you in assessing the properties of the items in the White Box, as well as the unopenable Black Box 1.3C.
- Add your results from the use of these tools to a new column in the tables you have made on the board at the front of the room.
- Has the additional examination of the problem using the new tools available to you changed the proportion of teams that feel a particular pair of items or a particular item is present in the Black Box 1.3C?

NOTE: While you are moving towards the truth, one cannot truly verify what is in Black Box 1.3C, as you are not able to open it and look inside. If you could, it would no longer be a black box, but a white box!

ANSWERS FOR THE BLACK BOX EXERCISE

Examples of hypothesis testing procedures

Procedure 1: Testing for each object separately

- First, place one object at a time in the white box tin and observe how the object alters the tin's characteristics. For example, you might note how heavy the tin feels with the object inside it. If you have a scale you could even weigh the tin with the object inside it. You could test to see if the magnet attracts the object through the tin. You could also try shaking the tin to see if the sound is loud or soft, sharp or dull. Try to think of other ways to examine the tin. Record your observations in a table like Table 1.

Table 1: The characteristics of the white box tin when it contains one object:

object	sound	weight	magnet
crayon			
paper clip			
marble			
cotton ball			
rubber ball			
penny			
poker chip			
cork			
toothpick			
metal ball			
rubber band			

- What is in the black box tin? Since there are only eleven possible objects, you can test the tin for each of them. This means that you will test eleven hypotheses of the form: "There is a ______ in the tin."
- For each hypothesis complete the following steps:

1) Use your table of observations to form predictions about what characteristics the tin should have if the hypothesis is true. For example, if your hypothesis is: "There is a marble in the black box tin," and you observed that the marble made a sharp metallic sound in the white box tin, then one of your predictions might be

that the black box tin will make a sharp metallic sound when you shake it.

2) Experiment with the black box tin to see if it satisfies your predictions.
3) If the black box tin fails to satisfy a prediction, then you can reject the hypothesis that you are testing, eliminate the hypothesized object from the list of possible objects, and begin to test a new hypothesis. Continuing with the example above, if you shake the black box tin and hear ***only*** a dull thud, then you should eliminate the marble from the list of possible objects and begin to test a new hypothesis.
4) If the black box tin satisfies all of your predictions, then the object you hypothesized was in the box should remain on the list and you can begin to test a new hypothesis.

- Continue to test the black box tin until you have tested it for each of the eleven objects.
- If more than two objects remain on your list of possibilities, you may want to think of some new experiments that could help you to eliminate more objects from the list.
- Once you have eliminated as many objects as you can from the list of possibilities, select two of the remaining objects as your final hypothesis.

Procedure 2: Testing the tin for each pair of objects

- Examine the weight, sounds and other characteristics of the black box tin, and record your observations in a table like Table 2.

Table 2: The characteristics of the black box tin

sound	weight	magnet

- What is in the black box tin? In this procedure you will test all possible pairs of objects to see if they are the pair that is in the black box tin. This means that you will test fifty-five hypotheses of the form: “There is a _____ and a _____ in the black box tin.”
- For each hypothesis complete the following steps:

1) Place the pair of objects that you hypothesize are in the black box tin box into the white box tin.
2) If your hypothesis is true, then the white tin box should now have the same characteristics as the black box tin, therefore you should predict that the white box tin will now have all of the characteristics that are listed in your table of observations. For example, if your hypothesis is: "There is a cork and a cotton ball in the black box tin," and you observed that the black box tin made a sharp metallic clink when you shook it, then one of your predictions might be that the white box tin will now also make a sharp metallic sound when you shake it. *Note that in this procedure your predictions are the same for each hypothesis.*
3) Experiment with the white box tin to see if it satisfies your predictions.
4) If the white box tin fails to satisfy any of your predictions then you can reject the hypothesis that you are testing, eliminate the hypothesized pair from the list of possible pairs, and begin to test a new hypothesis. Continuing with the example above, if you shake the white box tin and hear ***only*** a dull thud, then you should eliminate the cork and cotton ball from the list of possible pairs and begin to test a new hypothesis.
5) If the black box tin satisfies all of your predictions ,then the pair that you hypothesized was in the black box tin should remain on the list, and you should begin to test a new hypothesis.

- Continue to test the tin until you have tested it for each of the fifty-five pairs of objects.
- If more than one pair remains on your list of possibilities, you may want to think of some new experiments that could help you to eliminate more pairs from the list.
- Once you have eliminated as many pairs as you can from the list of possibilities, select one of the remaining pairs as your final hypothesis.

Q1. What is the probability of spherical (ball-shaped) object being in the Black Box? Express this probability in the five different ways described above. Remember, all of these values mean exactly the same thing!

There are four spherical objects that can possibly be in the Black Box (a marble, metal ball, rubber ball, and a cotton ball). Since there are 11 total possible objects that can be in the box, the probability of a spherical object being in the box is equal to:

$$4 \text{ out of } 11 = \mathbf{4{:}\,11} = \frac{\mathbf{4}}{\mathbf{11}} \approx \mathbf{0.36} \approx \mathbf{36\%}$$

Q2. What are the odds of a spherical object being in the Black Box?

There are 4 spherical objects that could be in the box, but there are 7 non-spherical objects that could be in the box. Therefore, the *odds* of a spherical object being in the box are 4:7.

Q3. If you were to simply guess the identity of the object in your Black Box without using elements of science to guide your interaction with the Black Box, what would be the probability that you got the answer correct?

If you only randomly guessed at the identity of the object in your Black Box without interacting with it in any way, the probability that you guessed the correct object would be

$$1 \text{ out of } 11 = \mathbf{1{:}\,11} = \frac{\mathbf{1}}{\mathbf{11}} \approx \mathbf{0.09} \approx \mathbf{9\%}$$

Hopefully the proportion of groups that guessed the correct object by using the scientific method is larger than this!

Q4. Suppose that you roll a six sided die. Let *E* denote the event that you roll less than a five. Write down all of the elements that belong to the event *E*.

E ={1, 2, 3, 4}

Q5. Let *B* be the event that you roll $\mathbf{1, 4}$, or $\mathbf{6}$, that is, let $\boldsymbol{B = \{1, 4, 6\}}$. Are *B* and *E* mutually exclusive? If not, which elementary events belong to both *B* and *E*?

B and *E* are not mutually exclusive, because both of them contain the elementary events {1} and {4}.

Q6. Suppose two coins are tossed. Let *A* be the event that the first coin is heads, and *B* be the event that the second coin is heads. Are *A* and *B* independent?

Events *A* and *B* are independent, because the flip of one coin has no effect on the result of the flip of the other.

Q7. Suppose that two children attend the same daycare. Let *A* be the event that the first child catches a cold and *B* be the event that the second child catches a cold. Are the events *A* and *B* independent?

In this case, events *A* and *B* are not independent. Since the students are in the same school, exposure to the first child, who has a cold, increases the probability that the second child catches a cold.

Q8. Suppose that you have a marble, a metal ball, and a penny. Imagine that you form a pair by choosing two objects from this set. How many ways can you form a pair?

There are six ways to form a pair if we consider the order that the objects are added: marble and metal ball, marble and penny, metal ball and penny, penny and metal ball, penny and marble, metal ball and marble.

Q9. How many *distinct* pairs can be formed from the three items above?

You should have noticed that though there are six different ways to form pairs from the three objects if order is considered, when only the identities of the objects are considered, there are only three distinct pairs of objects, as each unique pair is duplicated.

Q10. How many pairs in our Black Box experiment are in an event that has a penny?

Every object that isn't a penny can be used to make a pair with a penny. Since there are ten objects that are not pennies, there are ten pairs that have a penny: {penny, poker chip}, {penny, rubber ball}, {penny, rubber band}, {penny, metal ball}, {penny, marble}, {penny, tooth pick}, {penny, paper clip}, {penny, cotton ball}, {penny, cork}, {penny, crayon}.

Since there are ten pairs that have a penny, and 55 possible pairs, the probability that you choose a pair with a penny is:

$$\frac{\mathbf{10}}{\mathbf{55}} = \frac{\mathbf{2}}{\mathbf{11}} \approx \mathbf{0.18} \approx \mathbf{18\%}$$

Q11. What is the probability that your Black Box has a pair consisting of a penny or a marble, but not both a marble and a penny?

Since there are nine pairs that contain a penny but do not contain a marble, and nine pairs that contain a marble but do not contain a penny, there are eighteen pairs in this event. Since there are 55 possible pairs, the probability that this event occurs is

$$\frac{\mathbf{18}}{\mathbf{55}} \approx \mathbf{0.33} \approx \mathbf{33\%}$$

Q12. What is the probability that, by guessing alone, you correctly guess the identity of one (but not both) of the objects in your Black Box?

This is simply a generalization of **Q11**. Since the presence of a particular object in the tin is random with respect to other types of objects, the probability that you find a pair that contains one of the objects in your tin but not the other object of interest is the same as the probability that you find that you have a pair of objects in your tin with a marble or a penny, but not both, or

$$\frac{\mathbf{18}}{\mathbf{55}} \approx \mathbf{0.33}, \textbf{or 1 out 3 guesses}.$$

Q13. Fill in the following table regarding the number of possible shakes and cones in our hypothetical ice cream shop:

# of Scoops	# of possible shakes (no flavor duplication)	# of possible shakes (duplication allowed)	# of possible cones (no flavor duplication)	# of possible cones (duplication allowed)
1	5	5	5	5
2	10	15	20	25
3	10	35	60	125
4	5	70	120	625
5	1	126	120	3125

Q14. Regarding the Black Box experiment, are your hypotheses more related to combinations or permutations? Why?

In the Black Box experiment, your hypotheses are more related to combinations, because you are simply interested in the identities of each of the two mystery objects in your box, and not in the order in which they were placed into the box.

Q15. Using what you now know about combinations and permutations, and the fact that there is NO duplication of objects in the Black Box, calculate the number of different pairs of 11 objects from the White Box that could be placed into a Black Box.

Since the order in which each object of a pair does not matter, this is a combination. Since we also know that a pair of objects in the Black Box is comprised of two distinct objects (no duplicates), you can use the appropriate combination equation to calculate the answer as follows:

$$\frac{n!}{(n-r)!\,(r!)} = \frac{11!}{(11-2)!\,(2!)} = \frac{11!}{(9!)(2!)}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)(2 \times 1)} = \frac{39916800}{725760} = 55$$

Thus, there are a total of 55 different pairs of objects that could be in the Black Box.

Q16. Since the pairs of items placed into a Black Box are determined at random, what is the probability that there is a marble and a cork in any given Black Box?

Since there are a total of 55 possible pairs of objects that could be in the Black Box, and these are determined randomly, the probability that there is a marble and a cork in any given Black Box is

$$\frac{1}{55} \approx 0.018 \approx 1.8\%$$

Q17. What if you were given a Black Box and told that there were three items in the box (with the same restriction of no identical items). What would be the probability of correctly guessing *all three* items in the box without using the any elements of science, and without any contact with the Black Box whatsoever?

In order to find this answer, you first need to find out how many possible combinations of items could be in the Black Box:

$$\frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!\,(\boldsymbol{r}!)} = \frac{\mathbf{11}!}{(\mathbf{11}-\mathbf{3})!\,(\mathbf{3}!)} = \frac{\mathbf{11}!}{(\mathbf{8}!)(\mathbf{3}!)}$$

$$= \frac{\mathbf{11}\times\mathbf{10}\times\mathbf{9}\times\mathbf{8}\times\mathbf{7}\times\mathbf{6}\times\mathbf{5}\times\mathbf{4}\times\mathbf{3}\times\mathbf{2}\times\mathbf{1}}{(\mathbf{8}\times\mathbf{7}\times\mathbf{6}\times\mathbf{5}\times\mathbf{4}\times\mathbf{3}\times\mathbf{2}\times\mathbf{1})(\mathbf{3}\times\mathbf{2}\times\mathbf{1})} = \frac{\mathbf{39916800}}{\mathbf{241920}} = \mathbf{165}$$

Since there are a total of 165 possible combinations of three items (with no duplication), the probability of correctly guessing all three items in the box would be equal to

$$\frac{\mathbf{1}}{\mathbf{165}} \approx \mathbf{0.006} \approx \mathbf{0.6}\%$$